The effective g_A in the pf-shell

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We have calculated the Gamow-Teller matrix elements of 64 decays of nuclei in the mass range $A = 41$–50. In all the cases the valence space of the full pf-shell is used. Agreement with the experimental results demands the introduction of an average quenching factor, $q = 0.744 \pm 0.015$, slightly smaller but statistically compatible with the sd-shell value, thus indicating that the present number is close to the limit for large A.

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The observed Gamow Teller strength appears to be systematically smaller than what is theoretically expected on the basis of the model independent "$3(N-Z)$" sum rule. Much work has been devoted to the subject in the last fifteen years [1–4]. The heart of the problem can be summed up by defining the reduced transition probability as

$$B(GT) = \left(\frac{g_A}{g_V}\right)^2 \langle \boldsymbol{\sigma\tau} \rangle^2, \qquad \langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}_{\pm}^k || i \rangle}{\sqrt{2J_i + 1}}, \tag{1}$$

and asking: Is the observed quenching due to a renormalization of the g_A coupling constant —originating in non nucleonic effects— or is it the $\boldsymbol{\sigma\tau}$ operator that should be renormalized because of nuclear correlations?

The analysis of some pf-shell nuclei for which very precise data are available and full $0\hbar\omega$ calculations are possible, strongly suggests that most of the theoretically expected strength has been observed [5,6] . The quenching factor necessary to bring into agreement the calculated and measured values is directly related to the amplitude of the $0\hbar\omega$ model space components in the exact wave functions. This normalization factor can also be obtained from (d,p) or $(e,e'p)$ reactions and reflects the reduction in the discontinuity at the Fermi surface in a normal system. As such, it is a fundamental quantity, whose evolution with mass number is of interest.

In principle there are two ways of extracting it from Gamow Teller processes. One is to equate it to the fraction of strength seen in the resonance region in (p,n) reactions. The alternative is to calculate lifetimes for individual β decays and show that they correspond to the experimental values within a constant factor. The latter procedure is more precise, but demands high quality shell model calculations that until recently were available only up to $A = 40$ [7–9].

Our aim is to extend these analyses to the lower part of the pf shell. Full $0\hbar\omega$ diagonalizations are done using the ANTOINE code [10] with the effective interaction KB3, a minimally monopole modified version [11] of the original Kuo Brown matrix elements [12]. We refer to [13] for details of the shell model work.

Following ref. [14] we define quenching as follows: for beta decays populating well-defined isolated states in the daughter nucleus, the square root of the ratio of the experimental measured rate to the calculated rate in a full $0\hbar\omega$ calculation is called the quenching factor. An average quenching factor, q, implies an average over many transitions, and may be incorporated into an effective axial vector coupling constant:

$$q = \frac{g_{A,\mathrm{eff}}}{g_A}, \tag{2}$$

where g_A is the free-nucleon value of $-1.2599(25)$ [14]. Following ref. [7] we define

$$M(GT) = \left[(2J_i + 1)\, B(GT)\right]^{1/2}, \tag{3}$$

so as to have quantities independent of the direction of the transition. Note here that our reduced matrix elements follow Racah's convention [15]. In table I we list the $M(GT)$ values and compare them with the experimental results. The table contain all the transitions known experimentally. We also include the quantum numbers of the final states, the Q-values, the branching ratios and the experimental $\log ft$ values from which the $B(GT)$ values were obtained using

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$$(f_A + f^{\epsilon})t = \frac{6146 \pm 6}{(f_V/f_A)B(F) + B(GT)}. \quad (4)$$

the value 6.146 ± 6 is obtained from the nine best-known superallowed $0^+ \rightarrow 0^+$ decays [14]. f_V and f_A are the Fermi and Gamow-Teller phase-space factors, respectively [16,7]. f^{ϵ} is the phase-space for electron capture [17].

A quick look to the table shows that the calculated values are systematically larger than the experimental ones. In order to obtain the effective g_A, first we normalize the $M(GT)$ to the "expected" total strength, W (listed in table I) and defined by

$$W = \begin{cases} |g_A/g_V| \left[(2J_i+1)3|N_i - Z_i|\right]^{1/2} & \text{for } N_i \neq Z_i, \\ |g_A/g_V| \left[(2J_f+1)3|N_f - Z_f|\right]^{1/2} & \text{for } N_i = Z_i. \end{cases} \quad (5)$$

In figure 1 are plotted the experimental values versus the theoretical ones for

$$R(GT) = M(GT)/W. \quad (6)$$

The points follow nicely a straight line whose slope gives the average quenching factor, $q = 0.744 \pm 0.015$. Most $R(GT)$ values are much smaller than 1, reflecting the fact the strength in the decay window is small and fragmented. As a consequence, each individual decay may be sensitive to small uncertainties in the calculations, which can be averaged out by summing the total strength for each nucleus. Therefore we introduce a new quantity

$$T(GT) = \left[\sum_f R^2(GT, i \rightarrow f)\right]^{1/2}. \quad (7)$$

In the corresponding plot in figure 2 the points again follow closely the $q = 0.744$ line.

Comparing with the results in other regions, is suggestive

- pf shell, $q = 0.744 \pm 0.015$ this work,
- sd shell, $q = 0.77 \pm 0.02$ [8],
- p shell, $q = 0.82 \pm 0.015$ [9].

In the figures, both the lines for $q = 0.744$, and $q = 0.77$ are drawn and it is clear that there is not much to choose between them, and indeed, the average quenching factor of 0.77 has been extensively used in many pf-shell calculations, either in direct diagonalizations [13,5,6] or Shell Model Monte Carlo studies [18], leading to agreement with global Gamow-Teller strengths (as measured in (n,p) and (p,n) reactions) and lifetimes.

Nevertheless, the results in the three regions point to a decrease of q with mass number, and the closeness of the sd and pf values suggests that we have reached the large-A regime. This observation is quite consistent with the numbers extracted by Osterfeld from (p,n) data in heavier nuclei (see fig. 6 in [3]).

Whatever its origin, the q factor is a fundamental quantity telling us about correlations that are so well hidden, precisely behind overall renormalizations such as q, that their existence may be doubted.

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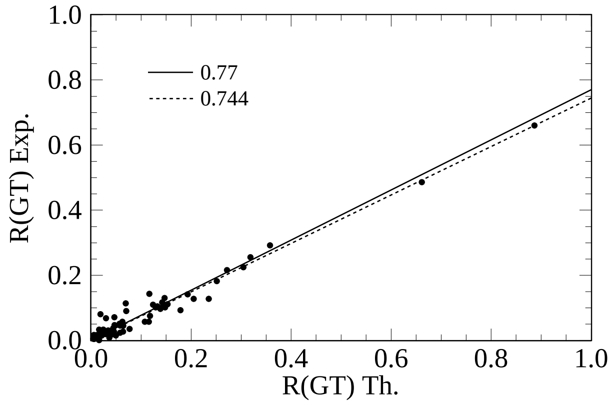

FIG. 1. Comparison of the experimental matrix elements $R(GT)$ with the theoretical calculations based on the "free-nucleon" Gamow-Teller operator. Each transition is indicated by a point in the x-y plane, with the theoretical value given by the x coordinate of the point and the experimental value by the y coordinate.

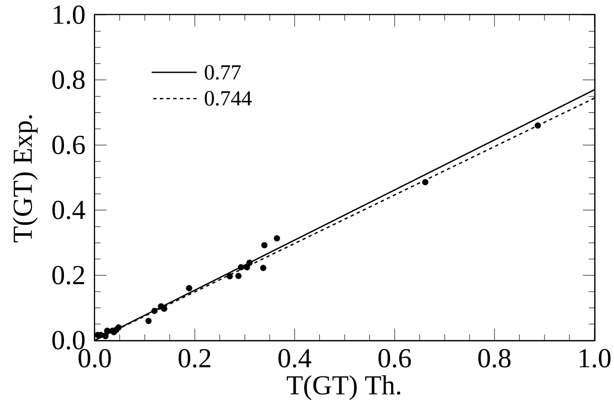

FIG. 2. Comparison of the experimental values of the sums $T(GT)$ with the correspondig theoretical value based on the "free-nucleon" Gamow-Teller operator. Each sum is indicated by a point in the x-y plane, with the theoretical value given by the x coordinate of the point and the experimental value by the y coordinate.

TABLE I. Experimental and theoretical $M(GT)$ matrix elements. The experimental data have been taken from [19]. $I_\beta + I_\epsilon$ are the branching ratios . All other quantities explained in the text.

Process	$2J_n^\pi, 2T_n^\pi$	Q (MeV)	$I_\beta + I_\epsilon$ (%)	$\log ft$	$M(GT)$ Exp.	$M(GT)$ Th.	W
^{41}Sc$(\beta^+)^{41}$Ca	$7^-, 1$	6.496	99.963(3)	3.461(7)	2.999	4.083	6.172
^{42}Sc$^*(\beta^+)^{42}$Ca	$12^+, 2$	3.851	100	4.17(2)	2.497	3.389	11.127
^{42}Ti$(\beta^+)^{42}$Sc	$2^+, 0$	6.392	55(14)	3.17(12)	2.038	2.736	3.086
^{43}Sc$(\beta^+)^{43}$Ca	$7^-, 3$	2.221	77.5(7)	5.03(2)	0.677	0.764	6.172
	$5^-, 3$	1.848	22.5(7)	4.97(3)	0.726	0.878	
^{44}Sc$(\beta^+)^{44}$Ca	$4_1^+, 4$	2.497	98.95(4)	5.30(2)	0.392	0.741	6.901
	$4_2^+, 4$	0.998	1.04(4)	5.15(3)	0.466	0.205	
	$4_3^+, 4$	0.353	0.010(2)	6.27(8)	0.128	0.295	
^{44}Sc$^*(\beta^+)^{44}$Ca	$12^+, 4$	0.640	1.20(7)	5.88(3)	0.324	0.276	11.127
^{45}Ca$(\beta^-)^{45}$Sc	$7^-, 3$	0.258	99.9981	5.983(1)	0.226	0.079	13.802
^{45}Ti$(\beta^+)^{45}$Sc	$7^-, 3$	2.066	99.685(17)	4.591(2)	1.123	1.551	6.172
	$5^-, 3$	1.342	0.154(12)	6.24(4)	0.168	0.280	
	$7^-, 3$	0.654	0.090(10)	5.81(5)	0.276	0.397	
	$9^-, 3$	0.400	0.054(5)	5.60(4)	0.351	0.712	
^{45}V$(\beta^+)^{45}$Ti	$7^-, 1$	7.133	95.7(15)	3.64(2)	1.801	2.208	6.172
	$5^-, 1$	7.093	4.3(15)	5.0(2)	0.701	0.428	
^{46}Sc$(\beta^-)^{46}$Ti	$8^+, 2$	0.357	99.9964(7)	6.200(3)	0.187	0.277	13.093
^{47}Ca$(\beta^-)^{47}$Sc	$7^-, 5$	1.992	19(10)	8.5(3)	0.012	0.262	16.331
	$5^-, 5$	0.695	81(10)	6.04(6)	0.212	0.235	
^{47}Sc$(\beta^-)^{47}$Ti	$5^-, 3$	0.600	31.6(6)	6.10(1)	0.198	0.235	13.802
	$7^-, 3$	0.441	68.4(6)	5.28(1)	0.508	0.611	

TABLE I. *Continuation.*

Process	$2J_n^\pi, 2T_n^\pi$	Q (MeV)	$I_\beta + I_\epsilon$ (%)	$\log ft$	$M(GT)$ Exp.	$M(GT)$ Th.	W
^{47}V$(\beta^+)^{47}$Ti	$5_1^-, 3$	2.928	99.552(15)	4.901(5)	0.555	0.896	4.365
	$3_1^-, 3$	1.378	0.049(6)	6.08(6)	0.143	0.107	
	$1_1^-, 3$	1.1337	0.285(10)	5.10(2)	0.442	0.563	
	$3_2^-, 3$	0.765	0.071(3)	5.36(2)	0.327	0.514	
	$5_2^-, 3$	0.761	0.0091(7)	6.25(4)	0.118	0.278	
	$5_3^-, 3$	0.402	0.0172(9)	5.41(3)	0.309	0.202	
	$3_3^-, 3$	0.379	0.0067(5)	5.77(4)	0.204	0.204	
	$1_2^-, 3$	0.134	0.0021(6)	5.18(9)	0.403	0.780	
^{47}Cr$(\beta^+)^{47}$V	$3^-, 1$	7.451	96.1(13)	3.70(2)	0.942	1.186	4.365
	$5^-, 1$	7.363	3.9(13)	5.1(2)	0.442	0.646	
^{48}Sc$(\beta^-)^{48}$Ti	$12_1^+, 4$	0.661	90.0(3)	5.532(13)	0.484	0.780	22.256
	$12_2^+, 4$	0.485	9.85(9)	6.010(17)	0.279	0.331	
^{48}V$(\beta^+)^{48}$Ti	$8_1^+, 4$	1.719	89.0(9)	6.175(7)	0.192	0.345	9.259
	$6^+, 4$	0.791	3.33(7)	6.565(10)	0.123	0.090	
	$8_2^+, 4$	0.775	7.76(9)	6.180(6)	0.191	0.181	
^{48}Cr(EC)^{48}V	$2^+, 2$	1.233	100	4.294(7)	0.559	0.709	5.346
^{48}Mn$(\beta^+)^{48}$Cr	$8_1^+, 0$	11.670	6.5(25)	5.4(2)	0.469	0.527	9.259
	$8_2^+, 0$	9.101	10.1(24)	4.6(1)	1.178	2.179	
	$8_3^+, 0$	8.876	4.0(9)	5.0(1)	0.743	0.172	
	$8_4^+, 0$	8.497	8.0(7)	4.58(5)	1.206	1.361	
	$10_1^+, 0$	8.235	3.2(4)	4.90(6)	0.834	0.651	

TABLE I. *Continuation.*

Process	$2J_n^\pi, 2T_n^\pi$	Q (MeV)	$I_\beta + I_\epsilon$ (%)	$\log ft$	$M(GT)$ Exp.	$M(GT)$ Th.	W
^{49}Ca$(\beta^-)^{49}$Sc	$3^-, 7$	2.178	91.5(7)	5.075(4)	0.455	1.007	13.093
	$5_1^-, 7$	1.190	7.0(7)	5.12(5)	0.432	0.209	
	$1^-, 7$	0.769	0.66(7)	5.42(5)	0.306	0.757	
	$5_2^-, 7$	0.524	0.21(6)	5.3(2)	0.351	0.591	
^{49}Sc$(\beta^-)^{49}$Ti	$7^-, 5$	1.994	99.94(1)	5.71(1)	0.309	0.469	16.331
	$9^-, 5$	0.371	0.010(3)	6.9(2)	0.079	0.072	
	$5^-, 5$	0.232	0.05(1)	5.6(1)	0.351	0.389	
^{49}V(EC)^{49}Ti	$7^-, 5$	0.602	100	6.2(1)	0.176	0.130	10.691
^{49}Cr$(\beta^+)^{49}$V	$7^-, 3$	2.631	12(2)	5.6(1)	0.304	0.335	5.346
	$5_1^-, 3$	2.540	37(2)	5.02(2)	0.593	0.817	
	$3^-, 3$	2.478	50(2)	4.81(2)	0.755	1.033	
	$5_2^-, 3$	1.116	0.081(9)	5.80(4)	0.242	0.312	
	$3_2^-, 3$	0.969	0.028(6)	6.15(8)	0.161	0.182	
	$5_3^-, 3$	0.396	0.0011(2)	6.75(7)	0.081	0.264	
	$3_3^-, 3$	0.322	$1.9(7)\ 10^{-4}$	7.3(2)	0.043	0.195	
^{49}Mn$(\beta^+)^{49}$Cr	$5^-, 1$	7.715	93.6(26)	3.67(3)	1.364	1.704	5.346
	$7^-, 1$	7.443	6.4(26)	4.8(2)	0.764	0.623	
^{50}Ca$(\beta^-)^{50}$Sc	$2^+, 8$	3.118	99.0(13)	4.14(2)	0.667	0.956	6.901
^{50}Sc$(\beta^-)^{50}$Ti	$8^+, 6$	4.213	8.4(18)	6.7(1)	0.116	0.208	20.471
	$12^+, 8$	3.689	88.4(15)	5.39(1)	0.525	0.572	
	$8^+, 8$	2.741	0.58(4)	7.01(4)	0.081	0.125	
	$10^+, 8$	2.007	1.58(5)	5.99(2)	0.263	0.358	